Weidenmuller.[24] Nonradiative effects due to electron screening, nuclear electromagnetic form factor effects, K-capture competition and second forbidden matrix element corrections seem to be agreed upon.[32] All of these effects are included in Table I.

It is fair to say that a vector boson *can* explain the current muon lifetime discrepancy, although its mass should not be much greater than 500 MeV. If the indications from CERN[3] that $M \approx 1.3 M_p$ are confirmed, we can only conclude that a sizeable discrepancy still remains to be explained.

[32] L. Durand, L. F. Landowitz, and R. B. Marr, Phys. Rev. Letters 4, 620 (1960).

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Diffraction Width in Terms of f^0 and the Residue of the Pomeranchuk Pole*

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At a sharp resonance the phase of a Regge residue $\beta(t)$ should be essentially a multiple of 2π. The value it takes determines to a large extent the falloff rate of β and of the reduced residue $\gamma=\beta/\nu^\alpha$ for $t\leqslant 0$. If f^0 lies on the Pomeranchuk trajectory and if the phase there is 2π then it turns out that $\gamma(t)$ falls off exponentially for small $-t$ with a width comparable to the one deduced from the widths of the high-energy diffraction peaks, and for large $-t$, $\gamma(t)$ has a power fall-off. On the other hand, if the phase at f^0 remains small then the width is at least an order of magnitude larger. The latter case is indicated on the basis of the potential theory results. However, it is possible that the former may be a purely relativistic phenomenon peculiar to the Pomeranchuk pole in which case the Regge-pole hypothesis would be consistent with the high-energy experiments.

UNITARITY implies that a Regge-pole term

$$\beta(t)/l-\alpha(t) \tag{1}$$

near a sharp resonance at $t=t_0$, $\alpha_R(t_0)=l$ satisfies the relations $\beta_R\approx\Gamma$ (the width of the resonance) and $\beta_I\ll\beta_R$. Since β_R is positive at t_0, the latter condition implies that the phase of β must essentially be a multiple of 2π at a resonance.[1] This is a strong restriction on the phase; the value it takes, namely, ≈ 0, 2π, etc., determines to a large extent how fast β or rather the reduced residue γ (see below) falls off for $t<0$. The behavior in the negative t region is of some interest since it was pointed out recently[2] that if γ of the Pomeranchuk pole (P) showed a sharp diffraction-type fall off for small $-t$, then the Regge-pole approximation[3] may still be adequate in explaining the high-energy behavior of scattering amplitudes. The question of shrinkage or absence of it can then be understood in terms of appropriate linear combinations of P with other important poles.[2] We shall show below that if f^0 lies on the P trajectory and if the phase is 2π at t_f then one obtains an exponential type falloff for small $-t$, with a width comparable to the one observed experimentally, and a power falloff for large $-t$.[4] Potential theory results, however, indicate that near a resonance the phase should stay small and not approach 2π.[5] If this is assumed to be true also for P then we find that it is impossible for γ to achieve a diffraction-type behavior; the width turns out to be at least an order of magnitude larger than the experimental values. This would strongly suggest that the pole-hypothesis is inadequate and that perhaps other singularities in addition to the commonly assumed poles play an important role.

Consider elastic $\pi\pi$ scattering with s the square of the c.m. energy and t the square of the momentum transfer. We shall take BeV as the unit of mass. For large s, the contribution of a Regge pole with position $\alpha(t)$, to the scattering amplitude $A_I(s,t)$ is given by

$$A_I(s,t)=-\rho_I\frac{2^{\alpha-1}(\pi)^{1/2}(2\alpha+1)\Gamma(\frac{1}{2}+\alpha)}{\Gamma(1+\alpha)}\times\left(\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\gamma(t)\left(\frac{s}{2M^2}\right)^\alpha, \tag{2}$$

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[1] More precisely, $2n\pi+O(\alpha_I)$. For a sharp resonance, $\alpha_I\ll 1$.

[2] B. R. Desai, Phys. Rev. Letters 11, 59 (1963).

[3] References to the earlier theoretical work are given in Ref. 2.

[4] Even if f^0 turns out to be 1^- or 3^- [see W. Frazer. S. Patil, and N. Xuong (to be published)] the essential points of this paper will not change provided that there exists a 2^+ particle on the P trajectory at a higher mass ($\lesssim 2$ BeV).

[5] A. Ahmadzadeh, thesis, University of California (unpublished). The results, based on Yukawa potentials, were communicated to me by G. F. Chew.

where

$$\gamma(t)=(M^2/\nu_t)^{\alpha(t)}\beta(t). \tag{3}$$

M is the nucleon mass, $t=4(\nu_t+m_\pi^2)$, I is the isotopic spin index, and ρ_I is the element of crossing matrix. For the P pole $\alpha(0)=1$ and $\rho_I=\frac{1}{3}$ for all $I(=0, 1, 2)$. From the optical theorem, $\gamma(0)=(0.06)\sigma_{\pi\pi}$, where $\sigma_{\pi\pi}$, in mb, should be about 15 from the factorization theorem.[6]

In the t channel, near the f^0 resonance, $t=t_f$, $\alpha_R(t_f)=2$, and the D wave should be well represented by the single term

$$\frac{\beta(t)/\alpha_R'(t)}{t_f-t-i[\alpha_I(t)/\alpha_R'(t)]} \tag{4}$$

if α_I is small, where

$$\alpha_I(t_f)=\Gamma_f(t_f)^{1/2}\alpha_R'(t_f) \tag{5}$$

and from unitarity,

$$\beta_R(t_f)=[t_f/(t_f-4m_\pi^2)]^{1/2}\alpha_I(t_f) \tag{6}$$

$$\beta_I(t_f)\ll\beta_R(t_f). \tag{7}$$

Γ_f is the width of f^0 and $\alpha_R'(t_f)$ is the slope of the P trajectory at t_f. Experimentally, $t_f=1.56$ and $\Gamma_f\approx0.20$. As a reasonable estimate we shall take $\alpha_R'(t_f)$ to be 1. It turns out that a change by a factor of 2 in the slope changes the diffraction width by not more than 10%.

The reduced residue $\gamma(t)$ is real along the real axis except for a cut from $t=4m_\pi^2$ to ∞. A phase representation can, therefore, be written as follows:

$$\gamma(t)=\gamma(0)e^{\eta(t)} \tag{8}$$

$$\eta(t)=\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{(t'-t)t'}\eta_I(t'), \tag{9}$$

where the phase η_I vanishes at threshold.[7,8] At infinity, on the basis of the strip-approximation result of Chew and Jones[9] the phase must approach $\epsilon\pi$, where $\epsilon\geqslant1$. Note that the expression (8) is arbitrary up to a finite number of zeros. The only strong evidence for a zero exists not for P, which is under consideration here, but for the ω pole.[10] Experimentally it is observed that at a given energy, $d\sigma/dt$ for $\bar{p}p$ at $t=0$ is much larger than that for pp and falls off very rapidly as $-t$ increases.[11,12] In fact, it goes below the pp value at around $-t\approx0.15$. A similar phenomenon occurs for K^-p-K^+p.[12] At the crossover point, therefore, $\gamma_\omega(t)$ should be zero since it occurs with different signs in $pp(K^+p)$ and $\bar{p}p(K^-p)$. Such zeros are possible if one has long-range attraction for ω followed by a short-range repulsion. As far as P is concerned, we shall, for the present, ignore the zeros.

An approximate analytical expression for $\eta_I(t)$ of P satisfying the above conditions is the following:

$$\eta_I(t)=\epsilon\pi[(t-4m_\pi^2)/(t+c)]^{\lambda_0},\quad t\geqslant4m_\pi^2,\quad \epsilon\geqslant1, \tag{10}$$

where $\lambda_0=\alpha(4m^2{}_\pi)+\frac{1}{2}$ and c is a positive constant. There is an additional logarithmic factor in (10) which has a negative sign near threshold but becomes positive eventually.[7] The effect of neglecting this factor on the width or on $\eta'(0)$ (see below) should not be large if at the same time we have $\eta_I(t_f)=2n\pi$ with $n\geqslant1$. The fact that this is a very large value attained at only moderately large energies means that the negative contribution of the logarithmic factor can come only from a narrow region near threshold where it will be suppressed by the strong threshold behavior ($\lambda_0>1$). We have no reason to believe that between threshold and f^0 there are any sign changes other than the one mentioned above. If they do occur and if the amplitudes of the oscillations are not very large, however, the same arguments can be applied as above to say that their contributions should be small. Furthermore, when η_I at f^0 is large the knowledge of $\eta_R(t_f)$ [see Eq. (11)] and of the behavior of η_I at threshold and infinity are strong enough to fix the value of $\eta'(0)$ within a small range. For the case where $\eta_I(t_f)$ is negligibly small, however, any sign changes will very likely reduce the value of $\eta'(0)$. The expression (10) in that case will allow us only to give an upper bound on $\eta'(0)$.

Since $\alpha'(0)$ is small (~0.2) on the basis of the πp data[2,11] we shall take $\alpha(4m_\pi^2)\approx\alpha(0)=1$. Thus, we have two relations deduced from (7) and (8):

$$\eta_R(t_f)=\ln\left[\frac{\gamma(t_f)}{\gamma(0)}\right]=P\frac{t_f}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{t'(t'-t_f)}\eta_I(t') \tag{11}$$

and[1]

$$\eta_I(t_f)\approx2n\pi,\quad n=0,\,1,\,\cdots, \tag{12}$$

which determine the values of ϵ and c.

One of the questions we are concerned with is whether $\gamma(t)$ has a sharp exponential-type behavior for small $-t$. If we denote by B the (width)$^{-1}$ of $d\sigma/dt$, then since $d\sigma/dt\sim\gamma^2(t)(s/2M^2)^{2[\alpha(t)-1]}$ and $s/2M^2=m_\pi M^{-2}E\simeq m_\pi E$, we obtain

$$B=2[\eta'(0)+\alpha'(0)\ln(m_\pi E)], \tag{13}$$

where E is the lab energy. One would expect B to be about $(2m_\pi)^{-2}\simeq10$ since the radius of interaction is about $(2m_\pi)^{-1}$. Experimental values for πp, pp, etc. are roughly of the same order.[11] Since $\alpha'(0)$ is small the second term in (13) in the energy interval of about 10–20 is quite small and therefore the major portion

[6] M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962); V. N. Gribov, and I. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).
[7] A. O. Barut and D. E. Zwanziger, Phys. Rev. 127, 974 (1962).
[8] R. G. Newton, J. Math. Phys. 3, 867 (1962).
[9] G. F. Chew and C. E. Jones, University of California Radiation Laboratory Report UCRL-10216 (unpublished).
[10] F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters 9, 183 (1962).
[11] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 10, 376 (1963); 10 543 (1963); 11, 425 (1963).
[12] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 503 (1963).

of the (width)$^{-1}$ should be contained in $\eta'(0)$. For large $-t$, $\gamma^2(t)$ behaves as $|t|^{-2\epsilon}$ and on the basis of the pp data, ϵ should be about 2.5.[13]

Among the many choices in (12) we shall consider the first two. It is clear that a larger phase at t_f implies more oscillations in $\gamma(t)$ for $t>4m_\pi^2$, and, therefore, if $\gamma(0)$ is held fixed, implies a sharper falloff for $t\leqslant 0$. If we take $\eta_I=2\pi$ at t_f, then $\eta'(0)$ is 4.9. The value of B obtained from (13) is then roughly what one observes in the high energy experiments. The value of ϵ is 3.4, not too different from the experimental results.[13] Also for phenomenological purposes, a good approximation to $\gamma(t)$ for $t\leqslant 0$ is found to be $\gamma(0)(1-t/t_0)^{-\epsilon}$, where $t_0(>4m_\pi^2)$ is essentially the point where η_R achieves its maximum value. In particular, $\epsilon\simeq 2.5$ and $t_0\simeq 0.5$ give a good fit to the diffraction data.

Note also that the value of $\eta'(0)$ and, therefore, of B is proportional to $\gamma(0)$, the total cross section. B depends inversely on Γ_f, the width of f^0. These properties are consistent with what one would normally expect of a function which gives the diffraction peak.

Consider now the case where η_I is small at t_f. For a sharp resonance, η_I should be roughly of the order of α_I. However, in order to obtain a firm upper bound, we shall take $\eta_I=\pi/4$ (i.e., $\beta_I=\beta_R$) at t_f. The value of $\eta'(0)$ then is 0.6 and ϵ is 1.2. Since these are overestimated values, the actual value of B obtained from (13) will be at least an order of magnitude smaller than the experimental results. Potential-theory results seem to favor this alternative.[5] The point is that if there is a sharp resonance then it is quite likely that (1) would satisfy unitarity for $l=\alpha_R(t_1)$, for t_1 up to the turning point of the trajectory.[14] By writing (1) in the Breit-Wigner form, we observe that since α_I remains small and positive, β_R should not be expected to change sign as t_1 increases from threshold towards the resonance region. Therefore, η_I should remain small at t_f, instead of approaching 2π.

In summarizing, let us consider the above results vis-à-vis the Regge-pole hypothesis. It was recently pointed out by Desai[2] that the pole approximation may still be adequate if the following assumptions are made: (i) $\alpha'(0)$ should be small in order to understand the absence of strong shrinkage in πp scattering. (ii) The experimentally observed sharp exponential fall off for small $-t$ and a power falloff for large $-t$ should then be given by $\gamma(t)$. (iii) The shrinkage in pp and K^+p should essentially come from the ω-pole which happens to be absent in πp. One of the predictions of this model, namely that the shrinkage in K^+p should be intermediate between πp and pp is borne out by the recent experiments.[12] As mentioned earlier, one will have to make the additional assumption of $\gamma_\omega(t)$ having a zero in order to explain the $\bar{p}p$ and K^-p scattering. But the most crucial assumption is (ii). If the potential theory results, mentioned above, hold also for the P trajectory, then $\gamma(t)$ will not have a sharp falloff and it would be impossible to understand the high-energy behavior in terms of poles alone. On the other hand, in the relativistic case, the effect of inelastic channels or nearby trajectories or perhaps cuts in the unitarity relation may enable β_R to change sign before reaching the resonance region. In that case the close agreement we have found with experiments will not be just a coincidence.

[13] R. Serber, Phys. Rev. Letters **10**, 357 (1963).

[14] G. F. Chew (private communication).